

Quantum decoherence of a charge qubit in a spin-fermion model

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We consider quantum decoherence in solid-state systems by studying the transverse dynamics of a single qubit interacting with a fermionic bath and driven by external pulses. Our interest is in investigating the extent to which the lost coherence can be restored by the application of external pulses to the qubit. We show that the qubit evolution under various pulse sequences can be mapped onto Keldysh path integrals. This approach allows a simple diagrammatic treatment of different bath excitation processes contributing to qubit decoherence. We apply this theory to the evolution of the qubit coupled to the Andreev-fluctuator bath in the context of widely studied superconducting qubits. We show that charge fluctuations within the Andreev-fluctuator model lead to a $1/f$ noise spectrum with a characteristic temperature dependence. We discuss the strategy for suppression of decoherence by the application of higher-order (beyond spin echo) pulse sequences.

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I. INTRODUCTION

The loss of coherence of a quantum two-level system (quantum bit) is caused by its unavoidable coupling to the surrounding environment. For solid-state qubits, the decoherence process can be quite fast due to coupling to a large number of internal degrees of freedom (DOFs). Our understanding of quantum decoherence and methods for its suppression in a realistic solid-state environment is mainly confined to the cases of a qubit interacting with bosonic¹⁻³ and nuclear spin baths,⁴⁻⁶ the so-called (and extensively studied) spin-boson and spin-bath models, respectively. A less well-understood but very relevant case for solid-state quantum architectures is that of a qubit coupled to a fermionic bath,⁷⁻¹³ which dramatically differs from the previous examples. In this paper we study the quantum decoherence in the context of a superconducting charge qubit¹⁴⁻²⁰ interacting with the nontrivial bath of Andreev fluctuators.²¹⁻²³ This problem is a paradigmatic spin-fermion decoherence problem and applies to many situations involving the quantum coupling of a qubit (“spin”) to a general fermionic environment. Using a many-body Keldysh path integral approach,^{9,10,24-26} we obtain a quantum-mechanical description of the qubit evolution under pulse sequences aimed at prolonging the coherence of the system. The simplest case of decoherence under pulses is the spin echo (SE) dephasing experiment, which has been shown to extend the coherence time of solid state (superconducting) qubits^{15,17,27} by essentially eliminating the quasistatic shifts of qubit energy splitting (inhomogeneous broadening) due to the slow environmental fluctuations. However, sequences involving more pulses, for example, Carr-Purcell-Meiboom-Gill (CPMG)²⁸ and Uhrig’s^{2,29} sequences, are expected to lead to a further increase in the coherence time.^{2,29-37}

In this paper, we consider an experimentally relevant example—a superconducting qubit coupled to fluctuating background charges,^{18,19} e.g., electrons residing on Anderson-impurity sites. Due to a large on-site Coulomb repulsion forbidding double occupancy, this example represents a nontrivial interacting bath. The dynamics of the

charge fluctuations on the impurity sites is determined by the hybridization of impurity levels with the quasiparticle band of the superconductor. To the lowest order in tunneling at the superconductor-insulator interface, the hybridization of the impurity levels can be described by a correlated tunneling event of two electrons with opposite spin to or from the superconductor. We show that in the small background-charge density limit, these fluctuations lead to a $1/f$ spectral density of noise. Using these results, we finally obtain the quantum-mechanical description of the qubit evolution driven by external pulses, and discuss an optimal strategy for the suppression of the decoherence with designed composite pulse sequences.

The paper is organized as follows: In Sec. II we provide a general derivation of the qubit evolution with pulses and map the calculation of decoherence function onto the Keldysh path integral formalism. In Secs. III and IV we introduce the Andreev-fluctuator bath and derive spectral density of noise for this model. Finally, in Sec. V we discuss the influence of pulse sequences on the qubit decoherence.

II. GENERAL THEORY FOR QUBIT EVOLUTION

The transverse dynamics of a qubit interacting with its environment is determined by the following Hamiltonian:

$$\hat{H} = \frac{E}{2} \hat{\sigma}_z + \hat{\sigma}_z \hat{V} + \hat{H}_B. \quad (1)$$

Here the environment is represented by a fermionic bath \hat{H}_B and the qubit is coupled to the environment through the density fluctuation operator,

$$\hat{V} = \sum_{l\sigma} v_l (c_{l\sigma}^\dagger c_{l\sigma} - \langle n_{l\sigma} \rangle). \quad (2)$$

This model corresponds precisely to the coupling of a superconducting charge qubit to the density fluctuations on the impurities in the substrate. Here $c_{l\sigma}$ and $c_{l\sigma}^\dagger$ are the fermionic annihilation, and creation operators at the l th site with spin σ and v_l and $\langle n_{l\sigma} \rangle$ are, respectively, the strength of the coupling

and average occupation of the l th impurity, i.e., $\langle n_{l\sigma} \rangle = \langle c_{l\sigma}^\dagger c_{l\sigma} \rangle$. Equations (1) and (2) define our spin-fermion model.

We study the evolution of the qubit in contact with a fermionic bath assuming the qubit energy relaxation time T_1 to be much longer than the quantum dephasing time T_2 (thus only $\hat{\sigma}_z$ coupling is present in the Hamiltonian). Qubit decoherence under the influence of the environment is given by the off-diagonal matrix elements of the qubit's reduced density matrix, and for the free evolution of the qubit we get ($\hbar=1$)

$$\rho_{+-}(t) = \langle + | \text{Tr}_B \{ \hat{\rho}(t) \} | - \rangle = \rho_{+-}(0) e^{-Et} W(t). \quad (3)$$

In the above $\hat{\rho}(t)$ is the density matrix of the whole system (qubit+bath), which is assumed to be factorizable at $t=0$, $\text{Tr}_B \{ \dots \}$ is the trace with respect to the bath DOFs, and $W(t)$ is the decoherence function defined as

$$W(t) = \langle e^{i(\hat{H}_B - \hat{V})t} e^{-i(\hat{H}_B + \hat{V})t} \rangle, \quad (4)$$

with the brackets representing the thermal average with respect to the bath Hamiltonian \hat{H}_B , i.e., $\langle \dots \rangle = \text{Tr}_B \{ \hat{\rho}_B \dots \}$. The time t always refers here to the total time of the evolution.

In addition to the free evolution of the qubit [free induction decay (FID)], one is often interested in the dynamics of the system subject to external π pulses^{2,29–37} which could, in principle, prolong or restore quantum coherence. The π pulses considered here correspond to the rotations of the qubit's Bloch vector by angle π about, e.g., the \hat{x} axis, and are short enough for the bath dynamics during the pulse duration to be negligible. Then, the evolution operator for qubit and bath is given by

$$\hat{U}^{(n)}(t) = (-i)^n e^{-i\hat{H}\tau_{n+1}} \hat{\sigma}_x e^{-i\hat{H}\tau_n} \dots \hat{\sigma}_x e^{-i\hat{H}\tau_1}, \quad (5)$$

with n and τ_i being the number of applied pulses and time delays between the pulses, respectively, and the total evolution time $t = \sum_{i=1}^{n+1} \tau_i$. One can see that the well-known Hahn SE sequence, for example, corresponds to a single pulse with $\tau_1 = \tau_2 = t/2$.

Using the fact that in the ‘‘pure dephasing’’ case under consideration, the qubit states $|\pm\rangle$ are the eigenstates of the Hamiltonian [(Eq. (1)), we can write the decoherence function under the influence of pulses as

$$W_n(t) = \langle (\hat{U}_-^{(n)}(t))^\dagger \hat{U}_+^{(n)}(t) \rangle, \quad (6)$$

with the evolution operators $\hat{U}_\pm^{(n)}(t)$ given by

$$\begin{aligned} \hat{U}_+^{(n)}(t) &= e^{-i(\hat{H}_B + \hat{V})\tau_{n+1}} e^{-i(\hat{H}_B - \hat{V})\tau_n} \dots e^{-i(\hat{H}_B + p\hat{V})\tau_1}, \\ \hat{U}_-^{(n)}(t) &= e^{-i(\hat{H}_B - \hat{V})\tau_{n+1}} e^{-i(\hat{H}_B + \hat{V})\tau_n} \dots e^{-i(\hat{H}_B - p\hat{V})\tau_1}, \end{aligned} \quad (7)$$

where $p = (-1)^n$ is the parity of the sequence. Then, the off-diagonal elements of the qubit density matrix are given by

$$\rho_{+-}(t) = \rho_{p,-p}(0) e^{-ipE(\tau_1 - \tau_2 + \dots + p\tau_{n+1})} W_n(t). \quad (8)$$

Here the phase factor is zero for all balanced sequences [for which the total times of evolution due to $\hat{H} + \hat{V}$ and $\hat{H} - \hat{V}$ are the same in Eq. (7)]. The evolution of the qubit under the SE

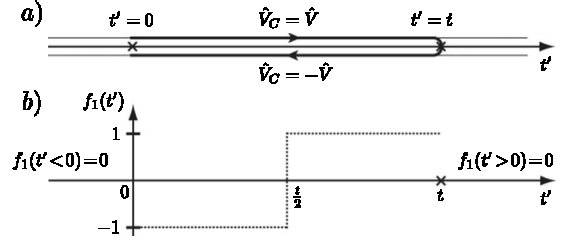


FIG. 1. (a) Dependence of $\hat{V}_C(t)$ on time along the Keldysh contour. (b) The plot of the function $f_n(t')$ for the SE sequence ($n=1$).

sequence, for example, acquires a simple form

$$\rho_{+-}^{\text{SE}}(t) = \rho_{-+}(0) \langle e^{i\hat{H}_+ t/2} e^{i\hat{H}_- t/2} e^{-i\hat{H}_+ t/2} e^{-i\hat{H}_- t/2} \rangle, \quad (9)$$

with $\hat{H}_\pm = \hat{H}_B \pm \hat{V}$.

Decoherence under pulses has been analyzed with methods specific to the spin-boson model^{2,29} and the spin-bath model,^{5,6} or using operator algebra.^{33,34,36} The latter approach, although very general, does not allow for transparent understanding of physics of the bath. However, the evaluation of $W_n(t)$ defined in Eq. (6) can be mapped onto the evolution on the Keldysh contour,³⁸ putting the calculation of decoherence into the framework of many-body theory. Similar formalism has been used to study full counting statistics of a general quantum-mechanical variable and has proved to be quite convenient.^{24,25} The evolution operators $\hat{U}_\pm^{(n)}$ can be written as

$$\hat{U}_\pm^{(n)}(t) = \mathcal{T} \exp \left\{ -i \int_0^t [\hat{H}_B \pm f_n(t') \hat{V}] dt' \right\}, \quad (10)$$

where \mathcal{T} is the time-ordering operator. The function $f_n(t')$ encodes a particular sequence and is defined as

$$f_n(t') = p \sum_{k=0}^n (-1)^k \Theta(t_{k+1} - t') \Theta(t' - t_k), \quad (11)$$

where $\Theta(t')$ is the Heaviside step function, t_k with $k=1, \dots, n$ are the times at which the pulses are applied, $t_0=0$, and $t_{n+1}=t$. Thus, the product of operators inside the average in Eq. (6) corresponds to (reading from left to right) the time-ordered evolution from 0 to t (with $+\hat{V}$ coupling), followed by the time-antioordered evolution from t to 0 (with $-\hat{V}$ coupling). We can then introduce the Keldysh contour C [see Fig. 1(a)] together with the notion of contour ordering of operators.^{26,38} The qubit-bath coupling takes then two opposite signs on the upper and lower branches of the contour: $\hat{V}_C = \pm \hat{V}$. While $f_n(t')$ is nonzero only for $t' \in [0, t]$, we can extend the limits of time integration on both branches to $[-\infty, \infty]$. The evolution from $t' = -\infty$ allows one to include the adiabatically turned-on interactions in \hat{H}_B (see, for example, Ref. 26), paving the way to the treatment of decoherence in an interacting fermionic bath. The final result is most compactly written as a functional integral with the Grassmann fields $\bar{\psi}_l$ and ψ_l defined on the Keldysh contour,^{9,10,26,38}

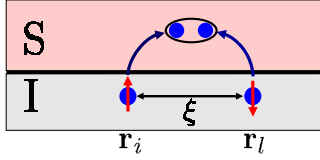


FIG. 2. (Color online) Correlated tunneling of two electrons with opposite spins from the impurity sites in the insulator into the superconductor. An electron from the i th impurity with energy below the gap Δ tunnels into the superconductor, propagates over distances of the order of coherence length ξ , and recombines with another electron with opposite spin from the j th site into a Cooper pair. The amplitude for such Andreev processes decays exponentially with distance between the impurity sites $A_{ij} \propto \exp(-|\mathbf{r}_i - \mathbf{r}_j|/\pi\xi)$, see Eq. (14).

$$\begin{aligned}
 W_n(t) &= \left\langle \mathcal{T}_C \exp\left(-i \int_C dt' [\hat{H}_B + \hat{V}_C f_n(t')]\right) \right\rangle \\
 &= \frac{1}{Z_B} \int \mathcal{D}\bar{\psi}_l \mathcal{D}\psi_l \exp\left(i S_B[\bar{\psi}, \psi] - i \int_C dt' \sum_{l\sigma} v_l(t') f_n(t') \right. \\
 &\quad \left. \times [\bar{\psi}_{l\sigma}(t') \psi_{l\sigma}(t') - \langle n_{l\sigma} \rangle]\right), \quad (12)
 \end{aligned}$$

where the integration is performed on the contour C shown in Fig. 1(a), $v_l(t') = \pm v_l$ on the upper and lower branches of the contour, and the normalization constant is defined as the functional integral with $\hat{V}=0$. The bath action $S_B = S_0 + S_{\text{int}}$, and the functional integration with noninteracting S_0 corresponds to averaging over an equilibrium noninteracting density matrix at $t' = -\infty$. This formulation of the decoherence problem enables one to use techniques and approximations developed in many-body theory. It also allows for a transparent treatment of the physics of the bath while simply encoding the driving of the qubit in a single function of time $f_n(t')$.

III. ANDREEV-FLUCTUATOR BATH

In order to evaluate the functional integral (12), one needs to specify the bath Hamiltonian. Here, as an example, we consider a nontrivial bath of Andreev fluctuators,^{21–23} which describes the fluctuations of the occupation of impurities close to the insulator-superconductor interface due to Andreev processes. This model takes into account coherent processes of creation (destruction) of the Cooper pair in the superconductor by correlated tunneling of two electrons from (to) different impurity sites in the insulator^{22,39} (see also Fig. 2). In the limit when the superconducting gap energy Δ is the largest relevant energy scale in the problem ($T, E, \varepsilon_j \ll \Delta$), the effective Hamiltonian for the Andreev-fluctuator bath, after integrating out superconducting DOFs, is given by

$$\hat{H}_B = \sum_{l\sigma} \varepsilon_l c_{l\sigma}^\dagger c_{l\sigma} + U \sum_l \hat{n}_{l\uparrow} \hat{n}_{l\downarrow} + \sum_{l \neq j} [A_{lj}^* c_{l\uparrow}^\dagger c_{j\downarrow}^\dagger + \text{H.c.}]. \quad (13)$$

Here, ε_l and U are the energies of a localized electron on the l th impurity (measured with respect to the Fermi energy ε_F

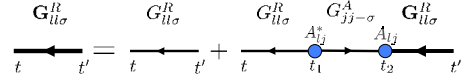


FIG. 3. (Color online) Dyson's equation for the retarded Green's function \hat{G}^R in the Born approximation. Here we have adopted the convention of Ref. 38. The advanced and Keldysh Green's functions are obtained analogously resulting in Eqs. (18) and (19).

of the conduction electrons) and repulsive on-site interaction (assumed to be large enough to prevent double occupation of the sites), respectively. The matrix elements A_{ij} in the limit of the low transparency barrier between the insulator and superconductor are given by

$$A_{ij} \approx A_0 \frac{\sin(p_F |\mathbf{r}_i - \mathbf{r}_j|)}{p_F |\mathbf{r}_i - \mathbf{r}_j|} e^{-|\mathbf{r}_i - \mathbf{r}_j|/\pi\xi}. \quad (14)$$

Here p_F is the Fermi momentum, and ξ is the coherence length in a clean superconductor. The amplitude $A_0 = 2\pi^2 d^2 a N(0) T_0^2$ is determined by the tunneling matrix element between the insulator and superconductor T_0 , the normal density of states in the metal $N(0) = m p_F / \pi^2$, the localization length under the barrier d , and the size of the impurity wave function a .²²

Given the Hamiltonian [Eq. (13)], the action for the bath on the Keldysh contour can be written as

$$\begin{aligned}
 S_B[\bar{\psi}, \psi] &= \sum_{lj} \int_C dt' \sum_{\sigma} \delta_{lj} \bar{\psi}_{l\sigma}(t') (i\partial_{t'} - \varepsilon_l - U \langle n_{l,-\sigma} \rangle) \psi_{l\sigma}(t') \\
 &\quad + A_{ij}^* \bar{\psi}_{l\uparrow}(t') \bar{\psi}_{j\downarrow}(t') + A_{ij} \psi_{j\downarrow}(t') \psi_{l\uparrow}(t'). \quad (15)
 \end{aligned}$$

Here we used the mean-field approximation for the Anderson-impurity model assuming that the Kondo temperature T_K is smaller than the superconducting gap Δ , which is reasonable in the situation at hand when the impurities are located in the substrate and the tunneling matrix element T_0 coupling them to the states in the superconductor is small. The occupation probabilities $\langle n_{l\sigma} \rangle$ are obtained self-consistently using

$$\langle n_{l\sigma} \rangle = \int \frac{d\omega}{2\pi} n_F(\omega) [G_{ll\sigma}^A(\omega) - G_{ll\sigma}^R(\omega)] \quad (16)$$

(see Ref. 40 for more details). Performing a Keldysh rotation,^{26,38} one can calculate the full Green's function $\hat{G}_{ll\sigma}^{\hat{G}}(t, t')$ for the bath (see Fig. 3),

$$\hat{G}_{ll\sigma}^{-1}(t, t') = \hat{G}_{ll\sigma}^{-1}(t, t') - \hat{\Sigma}_{ll\sigma}(t, t'). \quad (17)$$

Here $\hat{G}_{ll\sigma}^{-1}(t, t')$ is the bare Green's function [see Eq. (15)], and the self-energy $\hat{\Sigma}_{ll\sigma}(t, t')$ is calculated to the second order in A_{ij} giving the components of the self-energy matrix

$$\Sigma_{ll\sigma}^{A/R}(t, t') = \sum_{j \neq l} |A_{lj}|^2 G_{jj, -\sigma}^{R/A}(t', t), \quad (18)$$

$$\Sigma_{ll\sigma}^K(t, t') = \sum_{j \neq l} |A_{lj}|^2 G_{jj, -\sigma}^K(t', t). \quad (19)$$

In Eqs. (18) and (19) we have neglected the off-diagonal terms in the impurity indices, i.e., $\Sigma_{lj\sigma} \approx \delta_{lj} \Sigma_{ll\sigma}$. Since the amplitude A_{lj} oscillates on the length scale of p_F^{-1} , the contribution of these off-diagonal terms to the self-energy is small.

Using the above results, the action for the bath can be written in terms of the full Green's function $\hat{\mathbf{G}}_{ll\sigma}(t, t')$. Then, the decoherence function becomes

$$\begin{aligned} W_n(t) &\equiv \exp[-\chi_n(t)] \\ &= \frac{1}{Z_B} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp \left[i \sum_{l\sigma} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \right. \\ &\quad \times \left(\sum_{a,b=1}^2 \bar{\psi}_{l\sigma}^{(a)}(t_1) [\hat{\mathbf{G}}_{ll\sigma}^{-1}(t_1, t_2)]_{ab} \psi_{l\sigma}^{(b)}(t_2) \right. \\ &\quad \left. \left. - 2\delta(t_1 - t_2) v_l f^{(n)}(t_1) [\rho_{l\sigma}(t_1) - \langle n_{l\sigma} \rangle] \right) \right]. \quad (20) \end{aligned}$$

Here, $\rho_{l\sigma}(t)$ corresponds to the fermion density operator $\rho_{l\sigma}(t) = \frac{1}{2} [\bar{\psi}_{l\sigma}^{(1)}(t) \psi_{l\sigma}^{(2)}(t) + \bar{\psi}_{l\sigma}^{(2)}(t) \psi_{l\sigma}^{(1)}(t)]$; the fields $\psi^{(1)}(t)$ and $\psi^{(2)}(t)$ are given by the appropriate superposition of the fermionic fields on the upper and lower parts of the Keldysh contour, see Ref. 26. After performing the functional integral over the fermionic fields and expanding to the second order in v_l , one finds

$$\begin{aligned} \chi_n(t) &= \sum_{l\sigma} \frac{v_l^2}{2} \int_0^t \int_0^t dt_1 dt_2 f_n(t_1) f_n(t_2) [\mathbf{G}_{ll\sigma}^A(t_1, t_2) \mathbf{G}_{ll\sigma}^R(t_2, t_1) \\ &\quad + \mathbf{G}_{ll\sigma}^R(t_1, t_2) \mathbf{G}_{ll\sigma}^A(t_2, t_1) + \mathbf{G}_{ll\sigma}^K(t_1, t_2) \mathbf{G}_{ll\sigma}^K(t_2, t_1)]. \quad (21) \end{aligned}$$

Equation (21) holds whenever the short-time expansion is valid. The long-time asymptote can be obtained by resumming the whole series.^{9,10}

By introducing the Fourier transform of the Green's functions, Eq. (21) can be formally recast as

$$\chi_n(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{F_n(\omega t)}{\omega^2} S_Q(\omega). \quad (22)$$

Here $F_n(\omega t) = \omega^2 |f_n(\omega)|^2 / 2$ is a sequence-specific filter function, the role of which we discuss in Sec. V. Thus, to the second order in v_i we obtain $\chi_n(t)$ having the same structure as in the case of a qubit coupled to the spin-boson bath or classical noise,^{2,29,37,41} i.e., $\chi_n(t)$ is the integral of the product of the environment-specific spectral density of noise $S_Q(\omega)$ and sequence-specific filter function $F_n(\omega t)$. The spectral density of quantum noise $S_Q(\omega)$ in the spin-fermion problem is given by

$$\begin{aligned} S_Q(\omega) &= \sum_{l\sigma} v_l^2 \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \left[\mathbf{G}_{ll\sigma}^A \left(\Omega + \frac{\omega}{2} \right) \mathbf{G}_{ll\sigma}^R \left(\Omega - \frac{\omega}{2} \right) \right. \\ &\quad + \mathbf{G}_{ll\sigma}^R \left(\Omega + \frac{\omega}{2} \right) \mathbf{G}_{ll\sigma}^A \left(\Omega - \frac{\omega}{2} \right) \\ &\quad \left. + \mathbf{G}_{ll\sigma}^K \left(\Omega + \frac{\omega}{2} \right) \mathbf{G}_{ll\sigma}^K \left(\Omega - \frac{\omega}{2} \right) \right]. \quad (23) \end{aligned}$$

In the frequency domain, the full Green's functions are

$$\begin{aligned} \mathbf{G}_{ll\sigma}^{A/R}(\omega) &= \frac{1}{\omega - \varepsilon_l - U \langle n_{l, -\sigma} \rangle - \Sigma_{ll\sigma}^{A/R}(\omega)}, \\ \mathbf{G}_{ll\sigma}^K(\omega) &= \tanh\left(\frac{\omega}{2T}\right) [\mathbf{G}_{ll\sigma}^R(\omega) - \mathbf{G}_{ll\sigma}^A(\omega)], \quad (24) \end{aligned}$$

where the self-energy $\Sigma_{ll\sigma}^{A/R}(\omega)$ is defined as

$$\Sigma_{ll\sigma}^{A/R}(\omega) = \sum_{j \neq l} \frac{|A_{lj}|^2}{\omega + \varepsilon_j + U \langle n_{j\sigma} \rangle \mp i\delta}. \quad (25)$$

Equation (23) defines the noise spectral density in the quantum-mechanical many-body language, and thus enables a direct calculation of decoherence in various situations, as we consider next.

IV. SPECTRAL DENSITY OF NOISE DUE TO ANDREEV FLUCTUATORS

In general, the solution for $S_Q(\omega)$ with many Andreev fluctuators can be obtained numerically by randomly generating the energies ε_l and positions \mathbf{r}_l of the impurities at the insulator-superconductor interface. The numerically obtained spectral density of noise $S_Q(\omega)$ for 50 fluctuators is shown in Fig. 4. At low frequencies, the noise power spectrum has $1/f$ dependence.

For ω and A_0 much smaller than the typical impurity level spacing $\delta\varepsilon$ and temperature T , the analytical solution for the spectral density of noise (23) is given by

$$S_Q(\omega) \approx \sum_{l\sigma} 4v_l^2 \left[1 - \tanh^2\left(\frac{\tilde{\varepsilon}_{l\sigma}}{2T}\right) \right] \frac{\gamma_{l\sigma}(\tilde{\varepsilon}_{l\sigma})}{\omega^2 + 4\gamma_{l\sigma}(\tilde{\varepsilon}_{l\sigma})^2}, \quad (26)$$

where $\tilde{\varepsilon}_{l\sigma} = \varepsilon_l + U \langle n_{l, -\sigma} \rangle$ and $\gamma_{l\sigma}(\tilde{\varepsilon}_{l\sigma}) = \text{Im} \Sigma_{ll\sigma}^A(\tilde{\varepsilon}_{l\sigma})$ is the broadening of the impurity energy levels due to Andreev processes. This broadening corresponds to the fluctuations of the impurity occupations changing the electrostatic environment of the qubit, and thus causing dephasing. From Eq. (26), one can see that $S_Q(\omega)$ is given by a sum of Lorentzians with different widths, which under proper distribution of γ_l gives rise to a $1/f$ noise spectrum (see below). Given that the charge density fluctuations via Andreev processes involve two impurities with energies of the order of T , the probability to find two such impurities is proportional to $(T/D)^2$ with D being the impurity energy bandwidth, and thus, $S_Q(\omega) \propto T^2$ at low frequencies as seen experimentally.^{18,19}

For the $1/f$ spectrum to arise from Eq. (26), the distribution of γ_l has to be lognormal. In order to have such a dis-

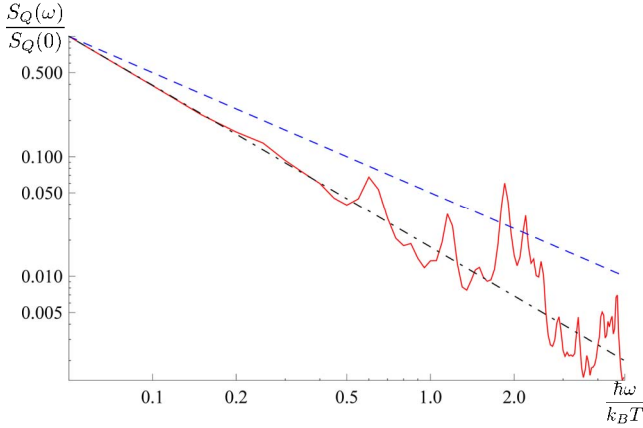


FIG. 4. (Color online) Log-log plot of the noise spectral density $S_Q(\omega)$ for Andreev-fluctuator model. The plot is obtained by randomly generating the positions $\mathbf{r}_l \in [-5, 5]\xi$ and energies $\varepsilon_l \in [-1, 1]$ K of 50 impurity sites and then numerically integrating Eq. (23). Here we assumed that $v_l = v$, on-site repulsion $U \rightarrow \infty$, and the sites are occupied with equal number of electrons with spins up and down. We used $p_F^{-1} = 10^{-2}\xi$, $A_0 = 0.1$ K, and $T = 0.1$ K. The dashed (blue) and dot-dashed (black) lines, shown for comparison, correspond to $1/f$ and $1/f^{1.3}$ noise spectra.

tribution, the density of the charge traps has to be small so that the dominant contribution to the self-energy in Eq. (25) comes from a few pairs of impurity sites, which are selected from the sum because of the energy conservation and distance constraint. Then, the switching rate $\gamma_l \propto \exp(-2|\mathbf{r}_l - \mathbf{r}_j|/\pi\xi)$ for a certain j [see Eq. (14)]. Since the distances between the charge traps are uniformly distributed, the probability of finding a switching rate γ is $P(\gamma) \propto 1/\gamma$, leading to $1/f$ noise. In the opposite limit of large density of charge traps, many sites j contribute to the sum in Eq. (25), and the switching rates γ_l self-average and become approximately the same for all sites. Note that unlike in the theory of $1/f$ charge noise produced by fluctuating two-level systems in the substrate with log-uniform distribution in the tunnel splitting,⁴² the emergence of the $1/f$ noise within Andreev-fluctuator model has a qualitatively different geometrical origin due to the exponential dependence of the rate γ_l on the distance between different impurity sites. This finding of the geometric origin of the $1/f$ noise in the Andreev-fluctuator model is an important result of our work.

We note that the model of charge traps with no on-site repulsion $U=0$ [Eq. (21)] does not lead to $1/f$ noise because in this case the self-energy is dominated by the two-electron tunneling from the same site. The contributions to the self-energy from Andreev processes involving other sites are exponentially smaller than the dominant term, and the distribution of the rates in Eq. (26) is not lognormal. Therefore, we emphasize that the realistic model for $1/f$ noise due to Andreev processes should include both spinful fermions (to correctly describe the dynamics of charge fluctuations), and large on-site repulsion (to prevent double-electron occupation).

At high frequencies $\omega \gg \delta\varepsilon, T$, the spectral density $S_Q(\omega)$ has resonances corresponding to the virtual processes of correlated two-electron tunneling from (to) the impurity sites in

the insulator. These resonances describing manifestly quantum-mechanical processes can be seen in Fig. 4 at high frequencies. Their contribution to the decoherence of the qubit is suppressed by a factor $F_n(\omega t)/\omega^2$, see Eq. (22). However, going beyond the pure dephasing model, $T_1 \gg T_2$, considered here, one can show that correlated two-electron tunneling processes contribute to the energy relaxation of the qubit.²¹

V. INFLUENCE OF PULSES ON DECOHERENCE

The time dependence of the decoherence function $W_n(t)$ under a pulse sequence is given by Eqs. (20)–(22), showing that the noise contribution is modulated by a filter function $F_n(\omega t)$. For the FID $F_0(\omega t) = 2 \sin^2[\omega t/2]$, whereas for SE we have $F_1(\omega t) = 8 \sin^4[\omega t/4]$ suppressing the low-frequency ($\omega \ll 4/t$) part of $S_Q(\omega)$. In general, higher-order pulse sequences act as more efficient high-pass filters of environmental noise, i.e., for n pulses applied in time t only frequencies $\omega > 2n/t$ contribute to $\chi_n(t)$. Due to the formal analogy between Eq. (22) and the solution for the decoherence under classical Gaussian noise, the analysis given for the latter case in Ref. 37 also applies here as long as the time expansion is valid. The results relevant for the noise spectral density derived here can be summarized as follows: For $S_Q(\omega) \propto 1/\omega^\alpha$, we obtain $\chi_n(t) \propto t^{1+\alpha}/n^\alpha$ for all sequences applicable to the pure dephasing case, i.e., the CPMG sequence,²⁸ periodic dynamical decoupling,³⁰ concatenations of SE,^{33,34} and Uhrig's sequence.² Thus, sequences beyond SE should lead to a further increase in coherence time for the $1/f$ spectral density of noise.^{32,35,37} For a noise spectrum without sharp ultraviolet cutoff, which is the case considered here, the CPMG sequence marginally outperforms other sequences.^{29,37} Furthermore, taking into account the simplicity of the CPMG sequence (defined by $\tau_1 = \tau_{n+1} = t/2n$ and all the other $\tau_i = t/n$), we believe that it is a preferred approach of noise suppression for the problem at hand.³⁷ We therefore propose that detailed experimental investigation of superconducting charge qubit dephasing behavior be carried out in order to test our specific predictions.

VI. CONCLUSION

We consider the spin-fermion model for quantum decoherence in solid-state qubits in the pure dephasing (i.e., $T_1 \gg T_2$) situation. We map the evolution of the qubit interacting with the fermionic environment, possibly subject to various π -pulse sequences onto the Keldysh path integral. This approach is very general and allows one to apply well-developed many-body techniques to the problem of the evolution of the qubit coupled to the environment and driven by pulses. In the short-time limit, we derive the expression for the qubit decoherence, which involves the product of the noise spectral density due to quantum fluctuations of the bath and the filter function representing a particular pulse sequence. For a nontrivial interacting model of the bath, the Andreev-fluctuator model, we show that the spectral density has $1/f$ dependence at low frequencies. Finally, we discuss the optimal strategy for the suppression of $1/f$ charge noise

by the application of higher-order (beyond spin echo) pulse sequences for the problem at hand. One of our concrete conclusions of experimental significance is that the well-established CPMG pulse sequence should be an optimal method for fighting T_2 dephasing when the noise spectrum has no sharp ultraviolet cutoff.

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